Background

Query Complexity

Communication Complexity

Proofs

Open Problems

A Stable Marriage Requires Communication

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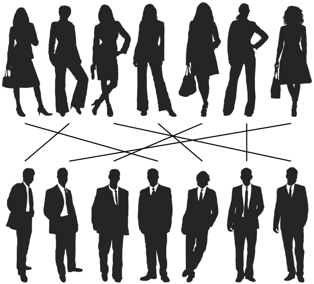

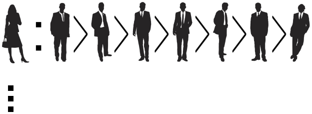

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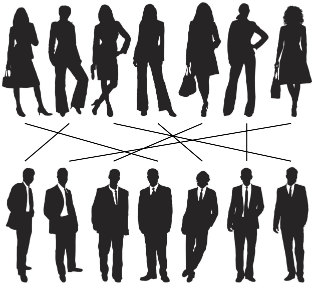

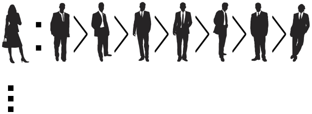

Roth (2002)

"Successful matching mechanisms produce stable outcomes."

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A stable marriage exists for every profile of preference lists.
A worst-case $\Theta(n^2)$-step algorithm for finding a stable marriage.

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3. When no more rejections occur, each woman marries the man serenading under her window. The resulting marriage is stable.

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Open Question (Gusfield, 1987)

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Partial Answers

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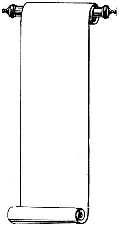

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Theorem (Chou and Lu, 2010)

If one is allowed to separately query each of the $\log n$ bits of the answer to queries such as "which man does woman w rank at place k?", then $\Theta(n^2 \log n)$ queries are still required.

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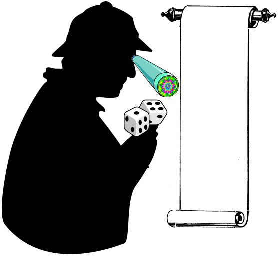

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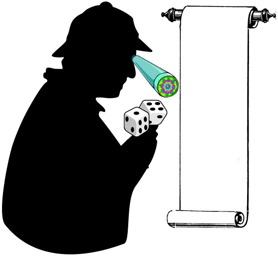

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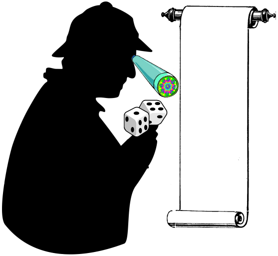

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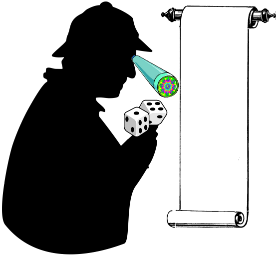

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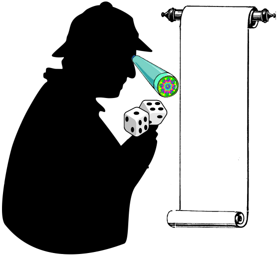

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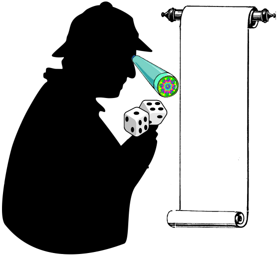

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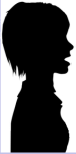

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- **Randomized** communication complexity is defined analogously using randomized protocols (with success rate bounded away from $\frac{1}{2}$).

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A lower bound on the communication complexity of each problem immediately implies the same lower bound on the number of Boolean queries in any algorithm for the same problem, even if arbitrary preprocessing of all the women's preferences and of all the men's preferences is allowed.

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Theorem (Segal, 2007)

Any deterministic communication protocol among all $2n$ participants for finding a stable marriage requires $\Omega(n^2)$ bits of communication.

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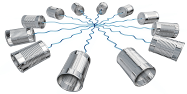

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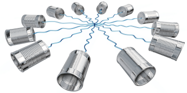


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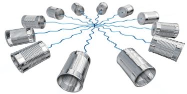

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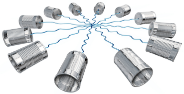

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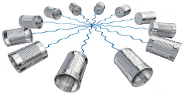

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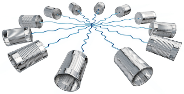

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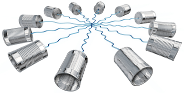

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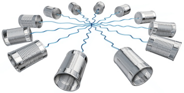

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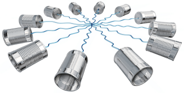

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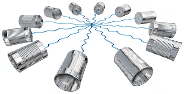

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Open Problem

Consider the comparison model for stable marriage that only allows for queries of the form "does man m prefer woman w_1 over woman w_2?" and, dually, "does woman w prefer man m_1 over man m_2?". How many such queries are required, in the worst case, to find a stable marriage?

Main Tool: The Disjointness Problem

Let $n \in \mathbb{N}$ and let $\bar{x} = (x_i)_{i=1}^n$ and $\bar{y} = (y_i)_{i=1}^n$ be bit vectors. The **disjointness** function is $\mathrm{DISJ}(\bar{x}, \bar{y}) \triangleq \neg \bigvee_{i=1}^n (x_i \wedge y_i)$, i.e., 0 if and only if there exists i s.t. $x_i = y_i = 1$.

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All of our results follow from defining suitable **embeddings** of DISJ into various problems regarding stable marriages, i.e., mapping $\bar{x}$ into preferences for the women and and $\bar{y}$ into preferences for the men, such that solving the stability-related problem reveals the value of $\mathrm{DISJ}(\bar{x}, \bar{y})$.

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Verifying / Finding an Exactly-Stable Marriage

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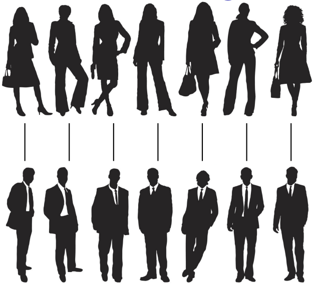

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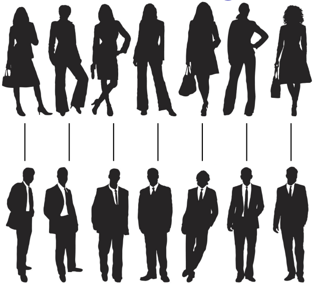

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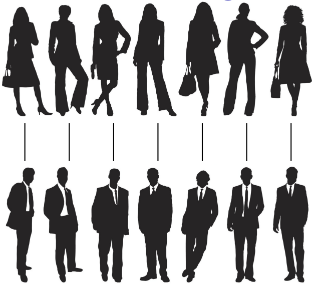

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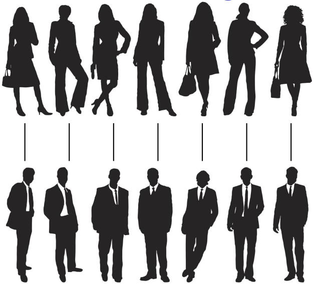

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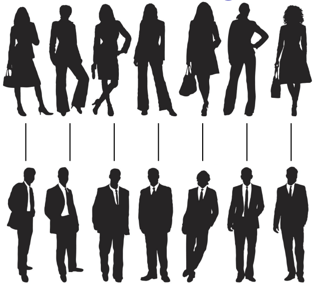

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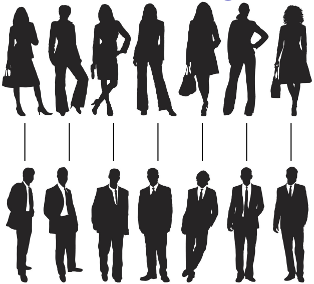

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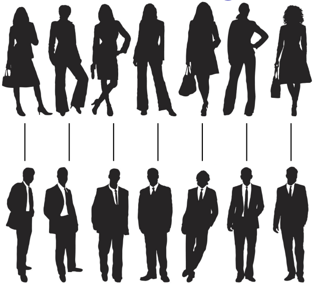

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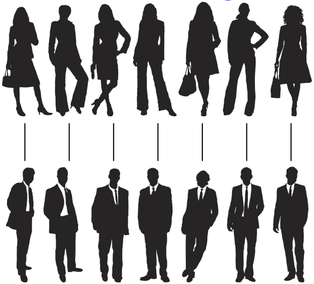

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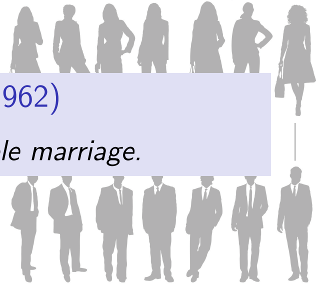
Theorem (Gale and Shapley, 1962)

There exists a men-optimal stable marriage.



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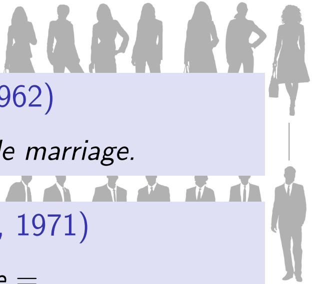

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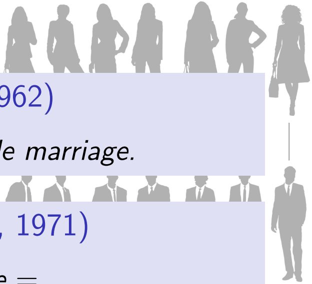

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Theorem (Rural Hospitals Theorem (Roth, 1984))

Each participant is either single in all stable marriages or married in all stable marriages.

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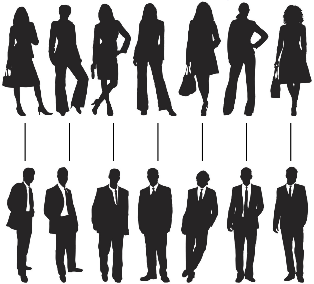

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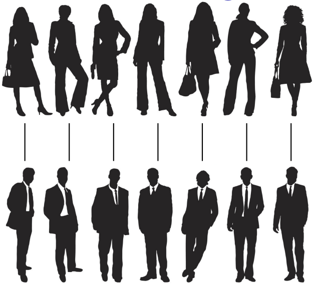

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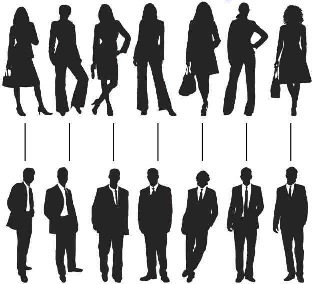

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- Color $\delta = \delta(\varepsilon)$ of the black suites in red. Let $\bar{x} = (x^i_j)_{i,j \in \{1,\ldots,\frac{\delta n}{2}\}}$ and $\bar{y} = (y^i_j)_{i,j \in \{1,\ldots,\frac{\delta n}{2}\}}$.

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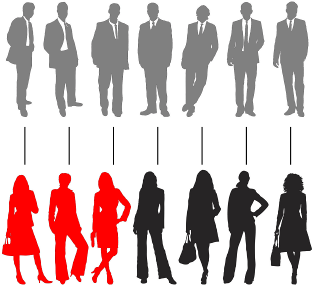

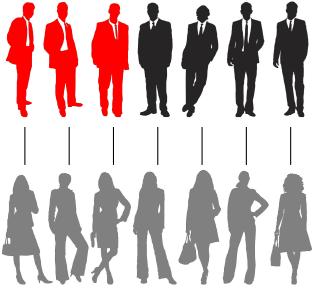

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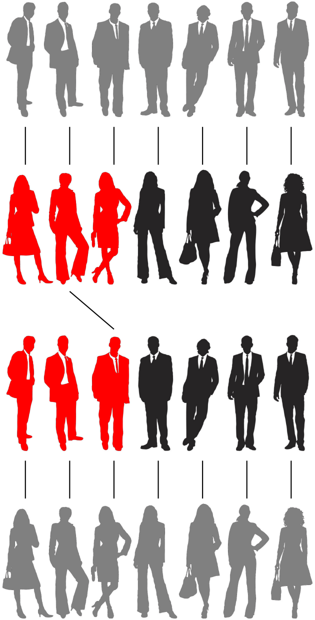

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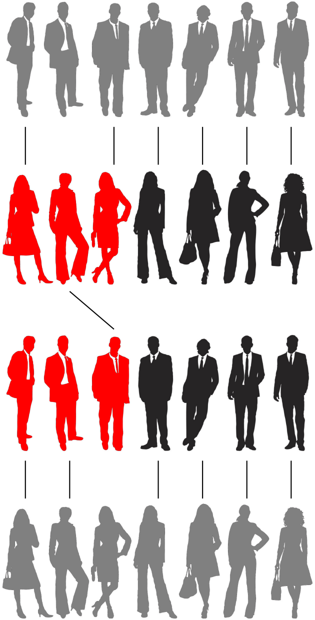

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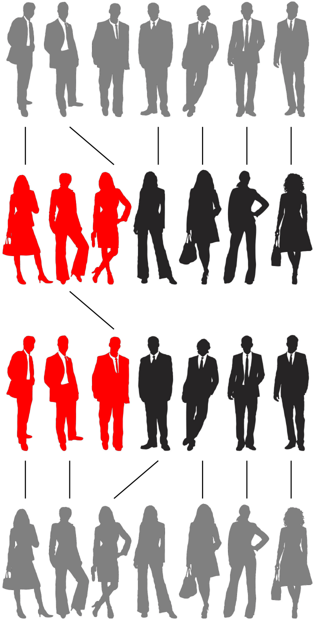

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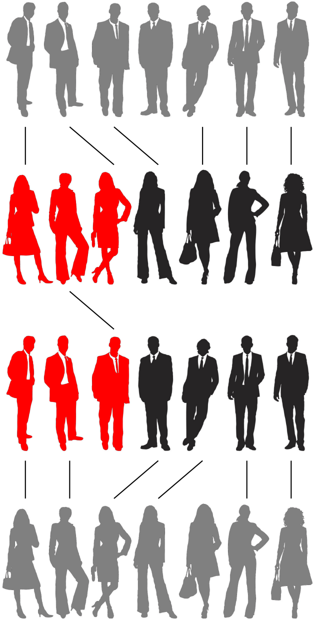

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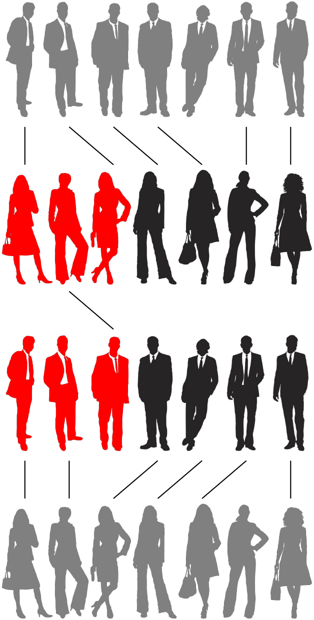

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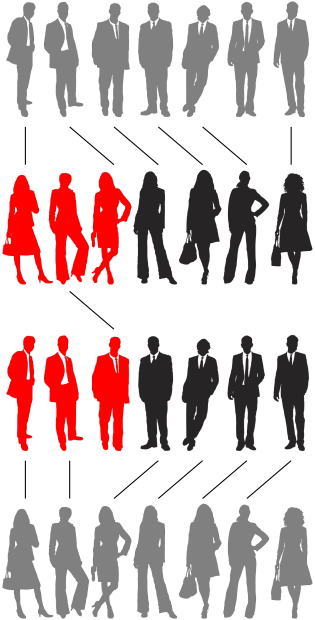

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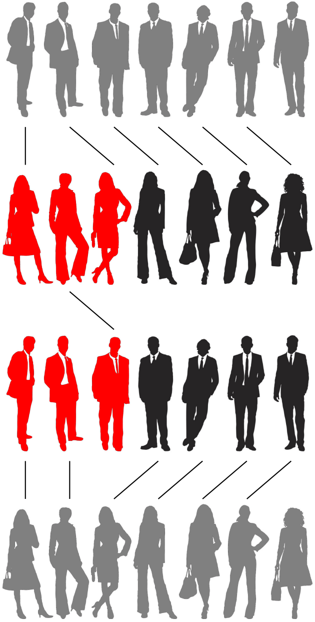

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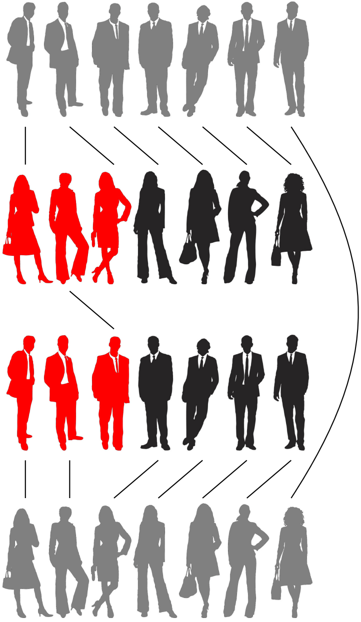

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- We have shown that finding a constant fraction of the pairs of a stable marriage is hard, and that so is verifying a single pair. What is the CC of finding a single pair that appears in some/every stable marriage?

Questions?

Thank you!